Factoring Overview

Is there a **Common Factor**?

Yes → Factor out the GCF

No

Are there 2 terms?

Yes → Is it a difference of squares? $a^2 - b^2$

Yes → $(a+b)(a-b)$

No → Is it a difference of cubes? $a^3 - b^3$

Yes → $(a-b)(a^2+ab+b^2)$

No → Is it a sum of cubes? $a^3 + b^3$

Yes → $(a+b)(a^2-ab+b^2)$

No → Prime

No

Are there 3 terms?

Yes → Is it a perfect square trinomial? $a^2+2ab+b^2$ or $a^2-2ab+b^2$

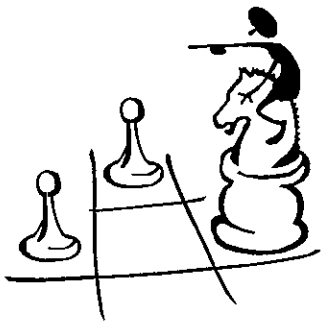
Yes → $(a+b)^2$ or $(a-b)^2$

No → Use trial and error or *ac* method

No

Are there 4 or more terms?

Yes → Use factoring by grouping



Examples

Factor:

1. $3x^2 - 12y^2$

$= \underline{3} \cdot x^2 - \underline{3} \cdot 4y^2$ ⟶ Factor out the GCF

$= 3(x^2 - 4y^2)$ ⟶ 2 terms are shown

$= 3[(x)^2 - (2y)^2]$ ⟶ Difference of two squares: $a^2 - b^2$

$= \underline{\underline{3(x-2y)(x+2y)}}$ ⟶ $(a-b)(a+b)$

2. $4x^3 + 32y^3$

$= \underline{4} \cdot x^3 + \underline{4} \cdot 8y^3$ ⟶ Factor out the GCF

$= 4(x^3 + 8y^3)$ ⟶ 2 terms are shown

$= 4[(x)^3 + (2y)^3]$ ⟶ Sum of cubes: $a^3 + b^3$

$= \underline{\underline{4(x+2y)(x^2 - 2xy + 4y^2)}}$ ⟶ $(a+b)(a^2 - ab + b^2)$

3. $8t^2 + 24t + 18$

$= \underline{2} \cdot 4t^2 + \underline{2} \cdot 12 \cdot t + \underline{2} \cdot 9$ ⟶ Factor out the GCF

$= 2(4t^2 + 12t + 9)$ ⟶ 3 terms are shown

$= 2[(2t)^2 + 2(2t \cdot 3) + 3^2]$ ⟶ Perfect square trinomial: $a^2 + 2ab + b^2$

$= \underline{\underline{2(2t+3)^2}}$ ⟶ $(a+b)^2$

4. $2x^2 + x - 15$

$a = 2, \qquad c = -15$

$a \cdot c = 2(-15) = -30$

Factors of -30	Sum of Factors
$-1 \cdot 30$	29
$-2 \cdot 15$	13
$-3 \cdot 10$	7
$\mathbf{-5 \cdot 6}$	**1**

$= 2x^2 - \underline{5}x + \underline{6}x - 15$

$= (2x^2 - 5x) + (6x - 15)$ ⟶ Factor by grouping

$= (2\underline{x}^2 - 5\underline{x}) + (\underline{3} \cdot 2 \cdot x - \underline{3} \cdot 5)$ ⟶ Factor out the GCF

$= x(\underline{2x - 5}) + 3(\underline{2x - 5})$ ⟶ Factor out the GCF

$= \underline{\underline{(x + 3)(2x - 5)}}$

5. $6x^2 - 5x - 4$□

$a = 6, \qquad c = -4$

$a \cdot c = 6(-4) = -24$

Factors of -24	Sum of Factors
$1(-24)$	-23
$2(-12)$	-10
$\mathbf{3(-8)}$	**-5**
$4(-6)$	-2

$= 6x^2 + \underline{3}x - \underline{8}x - 4$

$= (6x^2 + 3x) - (8x + 4)$ ⟶ Factor by grouping (be careful of negative sign!)

$= (\underline{3} \cdot 2\underline{x}^2 + \underline{3x}) - (\underline{4} \cdot 2x + \underline{4})$ ⟶ Factor out the GCF

$= 3x(2x + 1) - 4(2x + 1)$ ⟶ Factor out the GCF

$= \underline{\underline{(3x - 4)(2x + 1)}}$

6. $15x^2 + 18xy - 5xt - 6ty$

$= (15x^2 + 18xy) - (5xt + 6ty)$ ⟶ Factor by grouping (be careful of negative sign!)

$= (\underline{3} \cdot 5 \cdot \underline{x} \cdot x + \underline{3} \cdot 6 \cdot \underline{x} \cdot y) - (5 \cdot x \cdot \underline{t} + 6 \cdot \underline{t} \cdot y)$ ⟶ Factor out the GCF

$= 3x(\underline{5x + 6y}) - t(\underline{5x + 6y})$ ⟶ Factor out the GCF

$= \underline{\underline{(3x - t)(5x + 6y)}}$

Factoring Exercises

Greatest Common Factor

1. $18x-24$ **2.** $50x^5y^2+35x^2y$ **3.** $36a^6b+45a^5b^4+81a^3b^2$ **4.** $x(a+5)-y(a+5)$

Difference of Squares

5. t^2-25 **6.** $12x^2-27y^2$ **7.** $75A^2v-147t^2v$ **8.** $2x^4-512$

Difference of Cubes

9. p^3-64 **10.** $3a^3-24b^3$ **11.** $5x^3y-40y^4$ **12.** $125x^3-216y^6$

Sum of Cubes

13. $8a^3+64$ **14.** $128x^3+54$ **15.** $1000t^3u+27u$ **16.** $64x^3+343y^6$

Perfect Square Trinomials

17. $a^2+14a+49$ **18.** $16u^2+8u+1$ **19.** $t^2-12t+36$ **20.** $16x^2-24xy+9y^2$

ac Method

21. $x^2-4x-12$ **22.** $3k^2+4k-4$ **23.** $8a^2-10ab-3b^2$ **24.** $6m^2-19mn+10n^2$

Factor by Grouping

25. $6+3m+2p+mp$ **26.** $20+5s+12t+3st$ **27.** $4-2a-6b+3ab$ **28.** $5+x-5y-xy$

Factor:

29. $2x^3+128$ **30.** $4t^2-25$ **31.** $5a^3-45a^2+70a$

32. $12k^2-36k+27$ **33.** $a^3-b^3+2a-2b$ **34.** $15x^2+11xy-14y^2$

35. $2m^2-10m-48$ **36.** $72r^3s^2+12r^2-24r^4s^2$ **37.** $54y^2-24z^2$

38. $100n^2r^2+30nr^3-50n^2r$ **39.** $27p^{10}-45p^9-252p^8$ **40.** $16x^3z+2y^3z$

Solutions

1. $6(3x-4)$ **2.** $5x^2y(10x^3y+7)$ **3.** $9a^3b(4a^3+5a^2b^3+9b)$ **4.** $(x-y)(a+5)$

5. $(t+5)(t-5)$ **6.** $3(2x-3y)(2x+3y)$ **7.** $3v(5A-7t)(5A+7t)$ **8.** $2(x^2+16)(x+4)(x-4)$

9. $(p-4)(p^2+4p+16)$ **10.** $3(a-2b)(a^2+2ab+4b^2)$ **11.** $5y(x-2y)(x^2+2xy+4y^2)$

12. $(5x-6y^2)(25x^2+30xy^2+36y^4)$ **13.** $8(a+2)(a^2-2a+4)$ **14.** $2(4x+3)(16x^2-12x+9)$

15. $u(10t+3)(100t^2-30t+9)$ **16.** $(4x+7y^2)(16x^2-28xy^2+49y^4)$ **17.** $(a+7)^2$

18. $(4u+1)^2$ **19.** $(t-6)^2$ **20.** $(4x-3y)^2$ **21.** $(x-6)(x+2)$ **22.** $(3k-2)(k+2)$

23. $(4a+b)(2a-3b)$ **24.** $(3m-2n)(2m-5n)$ **25.** $(3+p)(2+m)$ **26.** $(4+s)(5+3t)$

27. $(2-3b)(2-a)$ **28.** $(5+x)(1-y)$ **29.** $2(x+4)(x^2-4x+16)$ **30.** $(2t+5)(2t-5)$

31. $5a(a-7)(a-2)$ **32.** $3(2k-3)^2$ **33.** $(a-b)(a^2+ab+b^2+2)$ **34.** $(5x+7y)(3x-2y)$

35. $2(m-8)(m+3)$ **36.** $12r^2(6rs^2+1-2r^2s^2)$ **37.** $6(3y+2z)(3y-2z)$

38. $10nr(10nr+3r^2-5n)$ **39.** $9p^8(3p+7)(p-4)$ **40.** $2z(2x+y)(4x^2-2xy+y^2)$